NAG Fortran Library Routine Document

G01GDF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G01GDF returns the probability associated with the lower tail of the non-central F or variance-ratio distribution, via the routine name.

2 Specification

```
real FUNCTION G01GDF(F, DF1, DF2, RLAMDA, TOL, MAXIT, IFAIL)
INTEGER              MAXIT, IFAIL
real                 F, DF1, DF2, RLAMDA, TOL
```

3 Description

The lower tail probability of the non-central F-distribution with ν_1 and ν_2 degrees of freedom and non-centrality parameter λ, $P(\mathrm{F} \leq f : \nu_1, \nu_2; \lambda)$, is defined by

$$P(F \leq f : \nu_1, \nu_2; \lambda) = \int_0^x p(F : \nu_1, \nu_2; \lambda) dF,$$

where

$$P(F : \nu_1, \nu_2; \lambda) = \sum_{j=0}^{\infty} e^{-\lambda/2} \frac{(\lambda/2)^j}{j!} \times \frac{(\nu_1 + 2j)^{(\nu_1+2j)/2} \nu_2^{\nu_2/2}}{B((\nu_1 + 2j)/2, \nu_2/2)}$$

$$\times u^{(\nu_1+2j-2)/2} [\nu_2 + (\nu_1 + 2j)u]^{-(\nu_1+2j+\nu_2)/2}$$

and $B(\cdot, \cdot)$ is the beta function.

The probability is computed by means of a transformation to a non-central beta distribution:

$$P(F \leq f : \nu_1, \nu_2; \lambda) = P_\beta(X \leq x : a, b; \lambda),$$

where $x = \dfrac{\nu_1 f}{\nu_1 f + \nu_2}$ and $P_\beta(X \leq x : a, b; \lambda)$ is the lower tail probability integral of the non-central beta distribution with parameters a, b, and λ.

If ν_2 is very large, greater than 10^6, then a χ^2 approximation is used.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: F – ***real*** *Input*

On entry: the deviate from the non-central F-distribution, f.

Constraint: $\mathrm{F} > 0$.

2: DF1 – ***real*** *Input*

On entry: the degrees of freedom of the numerator variance, ν_1.

Constraint: $0.0 < \text{DF1} \leq 10^6$.

3: DF2 – ***real*** *Input*

On entry: the degrees of freedom of the denominator variance, ν_2.

Constraint: $\text{DF2} > 0.0$.

4: RLAMDA – ***real*** *Input*

On entry: the non-centrality parameter, λ.

Constraint: $0.0 \leq \text{RLAMDA} \leq -2.0\log(U)$ where U is the safe range parameter as defined by X02AMF.

5: TOL – ***real*** *Input*

On entry: the relative accuracy required by the user in the results. If G01GDF is entered with TOL greater than or equal to 1.0 or less than $10\times$ ***machine precision*** (see X02AJF), then the value of $10\times$ ***machine precision*** is used instead.

6: MAXIT – INTEGER *Input*

On entry: the maximum number of iterations to be used.

Suggested value: 500. See G01GCF and G01GEF for further details.

Constraint: $\text{MAXIT} \geq 1$.

7: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: $\text{IFAIL} = 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output parameters may be useful even if $\text{IFAIL} \neq 0$ on exit, the recommended value is -1. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry $\text{IFAIL} = 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

If on exit $\text{IFAIL} = 1$ or 3, then G01GDF returns 0.0.

$\text{IFAIL} = 1$

On entry, $\text{DF1} \leq 0.0$,
or $\text{DF1} > 10^6$,
or $\text{DF2} \leq 0.0$,
or $\text{F} \leq 0.0$,
or $\text{RLAMDA} < 0.0$,
or $\text{MAXIT} < 1$,
or $\text{RLAMDA} > -2.0\log(U)$, where $U =$ safe range parameter as defined by X02AMF.

IFAIL = 2

The solution has failed to converge in MAXIT iterations. The user should try a larger value of MAXIT or TOL.

IFAIL = 3

The required probability cannot be computed accurately. This may happen if the result would be very close to 0.0 or 1.0. Alternatively the values of DF1 and F may be too large. In the latter case the user could try using a normal approximation; see Abramowitz and Stegun (1972).

IFAIL = 4

The required accuracy was not achieved when calculating the initial value of the central F (or χ^2) probability. The user should try a larger value of TOL. If the χ^2 approximation is being used then G01GDF returns zero otherwise the value returned should be an approximation to the correct value.

7 Accuracy

The relative accuracy should be as specified by TOL. For further details see G01GEF and G01GCF.

8 Further Comments

When both ν_1 and ν_2 are large a Normal approximation may be used and when only ν_1 is large a χ^2 approximation may be used. In both cases λ is required to be of the same order as ν_1. See Abramowitz and Stegun (1972) for further details.

9 Example

Values from, and degrees of freedom for, F-distributions are read, the lower-tail probabilities computed, and all these values printed, until the end of data is reached.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G01GDF Example Program Text
*     Mark 14 Release.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             DF1, DF2, F, PROB, RLAMDA, TOL
      INTEGER          IFAIL, MAXIT
*     .. External Functions ..
      real             G01GDF
      EXTERNAL         G01GDF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G01GDF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     F      DF1     DF2   RLAMDA   PROB'
      WRITE (NOUT,*)
      TOL = 0.5e-5
      MAXIT = 50
   20 READ (NIN,*,END=40) F, DF1, DF2, RLAMDA
      IFAIL = -1
*
      PROB = G01GDF(F,DF1,DF2,RLAMDA,TOL,MAXIT,IFAIL)
*
      IF (IFAIL.EQ.0) THEN
         WRITE (NOUT,99999) F, DF1, DF2, RLAMDA, PROB
```

```
      ELSE
         WRITE (NOUT,99999) F, DF1, DF2, RLAMDA, PROB,
     +     ' NOTE: IFAIL = ', IFAIL
      END IF
      GO TO 20
   40 STOP
*
99999 FORMAT (1X,4F8.3,F8.4,A,I1)
      END
```

9.2 Program Data

```
G01GDF Example Program Data
  5.5   1.5    25.5   3.0          :F DF1 DF2 RLAMDA
 39.9   1.0     1.0   2.0          :F DF1 DF2 RLAMDA
  2.5  20.25    1.0   0.0          :F DF1 DF2 RLAMDA
```

9.3 Program Results

```
 G01GDF Example Program Results

      F      DF1     DF2   RLAMDA   PROB

    5.500   1.500  25.500   3.000  0.8214
   39.900   1.000   1.000   2.000  0.8160
    2.500  20.250   1.000   0.000  0.5342
```